Stochastic Stability of Cooperative and Competitive Behavior in Cournot Oligopoly

Thijssen, J.J.J.

Publication date:
2001

No. 2001-19

STOCHASTIC STABILITY OF COOPERATIVE AND COMPETITIVE BEHAVIOUR IN COURNOT OLIGOPOLY

By Jacco Thijssen

March 2001

ISSN 0924-7815

Discussion paper

Stochastic Stability of Cooperative and Competitive Behaviour in Cournot Oligopoly*

Jacco Thijssen[†]

March 2001

Abstract

This paper considers a dynamic market where a fixed number of firms engage in Cournot competition. Firms are boundedly rational and are assumed to behave either competitively or cooperatively. In quantity setting firms are imitators. Firms may change their behaviour including social aspects in their decisions. It is shown that depending on the structure of the market and the social situation, either the Walrasian equilibrium or the collusion equilibrium is the unique stochastically stable state. Unlike the existing literature, this paper therewith gives a theoretical underpinning for some recent experimental results.

Keywords: Oligopoly theory, Stochastic Stability, Decomposability.
JEL codes: L13, C73.

1 Introduction

The economic decisions that an individual has to make are often very complicated. Each individual has to take into account not only her own objectives, actions and possible strategies, but also the social environment in which she operates. In the literature it has been argued that assuming rational behaviour by individuals, as is standard in the neo-classical paradigm, is

*I thank Dolf Talman for inspiring discussions and meticulous proof-reading. Helpful suggestions from Sem Borst and in particular Richard Boucherie are greatly appreciated. The usual disclaimer applies.

[†]Department of Econometrics & Operations Research and CentER, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands. E-mail: `J.J.J.Thijssen@kub.nl`.

too strong an assumption. To cite Young (1998): "This [rationality] is a rather extravagant and implausible model of human behavior, especially in the complex, dynamic environments that economic agents typically face". Furthermore, it is felt that the role of *equilibrium* in economic models needs to be reconsidered. One may look at the equilibria arising in neo-classical models as being the outcome of a dynamic process of interaction during which evolution to an equilibrium is established. Evolution then works as a way to select an equilibrium. The importance of equilibrium selection has also been recognized by Harsanyi and Selten (1988). They stick however to the neo-classical paradigm with rational individuals in an essentially static environment. Equilibrium selection then takes place by a rational procedure that can be described by a homotopy. Lately it has been argued that the equilibrium selection process should be seen as an evolutionary process and hence be modelled explicitely. The way this has been done is to apply ideas from evolution theory in game theory (see e.g. Maynard Smith (1982)). Evolution then results from the actions of essentially non-rational or boundedly rational agents. The main question is whether evolution leads in the long-run to a rational equilibrium, that is, do the equilibria arising in models based on the neo-classical paradigm give a good description of the long-run in an evolutionary setting. The field of evolutionary economics has grown rapidly the last decade. This development has been accompanied by an increasing interest in experimental economics to study the evolution of economic behaviour.

In evolutionary game theory, two concepts are considered to be standard notions. The first one is evolutionary stable strategy (ESS). An ESS is a strategy that is robust against small perturbations. As such it is a refinement of the Nash equilibrium concept (see e.g. van Damme (1994) or Weibull (1995)). To include an explicit evolutionary dynamic process the concept of stochastic stability has been introduced (see Foster and Young (1990), Kandori et al. (1993), and Young (1993)). Intuitively, stochastically stable states are those states that remain to be played after evolution has done its work. The argument is that as time goes along, economic agents learn to understand their environment better. The stochastically stable states are then the remaining states in the long-run that can be said to have survived the process of evolution.

In oligopoly theory these ideas have been applied by Vega-Redondo (1997). He showed that when firms imitate the firm that makes the highest profits, combined with experimentation or trial and error, in a Cournot setting the market converges to the Walrasian equilibrium. The importance of imitation and trial and error in evolutionary economic processes has already been stressed by Alchian (1950). He claimed that profit maximization is not a meaningful criterion for selecting an action in an essentially uncertain situation. Recently, an experimental study by Offerman et al. (2000) shows that in a Cournot market where agents know the demand function, the cost function, as well as all quantities produced and profits earned by the others, the market either settles on the Walrasian equilibrium or on the collusion equilibrium, i.e. the quantity that maximizes industry profit. The result by Vega-Redondo (1997) explains only part of this experimental result.

In this paper we allow that on a meta-level, individuals can change their behaviour. The evolution of behaviour leads to different evolution paths at the lower – quantity setting – level. We show that the stochastically stable state of this two-stage evolutionary process is either the collusive equilibrium resulting from cooperative behaviour or the Walrasian equilibrium resulting from competitive behaviour. Therewith, this paper gives a theoretical underpinning for the experimental results of Offerman et al. (2000). Which way evolution goes – competition or cooperation – depends on the inclination to cooperative behaviour within the group of firms in the market. This inclination can be seen as a social contract between firms and is therefore a representation of the social environment in which firms operate. It is assumed that the inclination to cooperation is exogenously given. The importance of the interrelationships of the environment and the prevailing types of economic behaviour which appear through a process of economic selection has been stressed by Alchian (1950). In the ultra-long run, the inclination to cooperative behaviour as reflected by the prevailing social contract might also be subjected to evolution. For a theory on the evolution of social contracts we refer to e.g. Binmore (1998). By allowing for the simultaneous existence of more than one behavioural rule and by introducing a meta-level where firms evaluate their behaviour, this paper extends the model of Vega-Redondo (1997) in two ways that, to our knowledge, have not been pursued so far in the existing literature.

In modeling the interaction on two levels a Markov chain on the quantity-setting level as well as a Markov chain on the behavioural level are defined, where the transition probabilities of the latter depend on the state of the quantity-setting chain. To get an approximation of the stochastically stable states of the entire process we use the theory of nearly completely decomposable systems developed by Simon and Ando (1961), Ando and Fisher (1963), and Courtois (1977). In the quantity-setting level, firms are assumed to imitate in a prescribed way. This corresponds to firms using rules of thumb in conducting their day to day business. On the behavioural level, firms reflect on their strategy and change their behaviour if the results of their current behaviour are not satisfactory.

Although the use of multiple behavioural rules in stochastic evolutionary models is new, the idea has been applied in an ESS setting, notably in the literature on indirect evolution (cf. Güth and Yaari (1992)). In this literature evolution works on preferences rather than on strategies. Possajennikov (2000) for example develops a model in which agents can be either altruistic or spitefull. Given the player's preferences she plays a best response. Altruism then leads to the cooperative outcome and spitefulness leads to the competetive outcome. Possajennikov (2000) gives conditions under which either altruism or spitefullness is evolutionary stable.

The paper is organized as follows. In Section 2 the model is explained in detail. First the quantity setting level is modeled. Afterwards the behavioural level is modeled and both levels are united in one stochastic dynamic system. In Section 3 it is proved that under suitable conditions either cooperative or competitive behaviour is stochastically stable. This leads to the conclusion that (approximately) either the collusion equilibrium or the Walrasian equilibrium arises. Finally, Section 4 concludes.

2 The Model

Let be given a market for a homogeneous good with n firms, indexed by $I_n = \{1, 2, \ldots, n\}$. Competition takes place by quantity setting. The profit of firm i, $i \in I_n$ if it produces γ_i and the other firms produce together

$\gamma_{-i} := \sum_{j \neq i} \gamma_j$ is given by

$$\pi(\gamma_i, \gamma_{-i}) = P(\gamma_i + \gamma_{-i})\gamma_i - C(\gamma_i), \tag{1}$$

where $P : \mathbb{R}_+ \to \mathbb{R}_+$ is the inverse demand function, which is assumed to be strictly decreasing and $C : \mathbb{R}_+ \to \mathbb{R}_+$ is the cost function. It is assumed that each firm can choose from a finite discrete set of possible production levels, $\Gamma(\delta) := \{0, \delta, 2\delta, \dots, v\delta\}$, for some $v \in \mathbb{N}$ and $\delta > 0$.

The behaviour of firm $i \in I_n$ is denoted by $\theta_i \in \Theta = \{0, 1\}$, where $\theta_i = 1$ denotes cooperative behaviour and $\theta_i = 0$ denotes competitive behaviour. The state-space is given by $\Omega = \Gamma(\delta)^n \times \Theta^n$. A typical element $\omega \in \Omega$ is denoted $\omega = (\gamma, \theta)$, where $\gamma = (\gamma_1, \dots, \gamma_n) \in \Gamma(\delta)^n$ and $\theta = (\theta_1, \dots, \theta_n) \in \Theta^n$. The cardinality of Θ^n will be denoted by N and the cardinalty of $\Gamma(\delta)^n$ by m. Let $\Sigma(S)$ denote the ordered set of all permutations on a finite set S. Then $\gamma(k) = \Sigma(\Gamma(\delta)^n)_k$, $k = 1, \dots, m$, and $\theta(I) = \Sigma(\Theta^n)_I$, $I = 1, \dots, N$.

2.1 The quantity setting level

Firms that behave cooperatively are assumed to imitate the exemplary firm, i.e. the firm that produces the output that generates the highest industry profits were all firms to produce the same amount. Therefore, these firms are called *exemplary imitators*. Given a vector of output levels $\gamma \in \Gamma(\delta)^n$ their best-response correspondence is given by

$$\begin{aligned} BE(\gamma) =& \{q \in \Gamma(\delta) | \exists_{j \in I_n} : q = \gamma_j, \forall_{k \in I_n} : \pi(q, (n-1)q) \\ & \geq \pi(\gamma_k, (n-1)\gamma_k)\}. \end{aligned} \tag{2}$$

If a firm acts competitively, it imitates the firm that makes the highest profit. These firms are called *profit imitators*. Given a vector of output levels $\gamma \in \Gamma(\delta)^n$ their best-response correspondence is given by

$$\begin{aligned} BP(\gamma) =& \{q \in \Gamma(\delta) | \exists_{j \in I_n} : q = \gamma_j, \forall_{k \in I_n} : \pi(q, \gamma_{-j}) \\ & \geq \pi(\gamma_k, \gamma_{-k})\}. \end{aligned} \tag{3}$$

Let $\theta(I)$ be a given permutation on Θ^n. Firm i chooses an output level from $BE(\gamma)$ if $\theta(I)_i = 1$. If $\theta(I)_i = 0$, firm i chooses an element from $BP(\gamma)$. To do so, firm i uses a probability distribution with full support, $\eta^{\gamma}_{E,i}(\cdot)$ if $\theta(I)_i = 1$ and $\eta^{\gamma}_{P,i}(\cdot)$ if $\theta(I)_i = 0$. The market now works as follows. At any time t, $t \in \mathbb{N}$ every firm gets the opportunity to adapt its output with a positive probability p. Given the vector of output levels $\gamma^{t-1} \in \Gamma(\delta)^n$, if firm i is a profit imitator then $\gamma_i^t \in BP(\gamma^{t-1})$, and if firm i is an exemplary imitator then $\gamma_i^t \in BE(\gamma^{t-1})$. These dynamics define a Markov chain on $\Gamma(\delta)^n \times \Gamma(\delta)^n$, with transition matrix M_0^I. The dynamic process evolving according to this Markov chain is called the *pure imitation dynamics*.

With probability $\varepsilon > 0$ firm i makes a mistake and chooses any output level using a probability distribution $\nu_i(\cdot)$ with full support. This can also be seen as experimentation or, to stay close to the original biological setting, mutation. Imitation and experimentation define a Markov chain on $\Gamma(\delta)^n \times \Gamma(\delta)^n$ with transition matrix M_ε^I. A typical element is given by

$$M_\varepsilon^I(k,l) = \prod_{i \in I_n} \Big\{ (1-\varepsilon)\big[p 1_{(\theta(I)_i=0)} \eta_{P,i}^{\gamma(k)}(\gamma(l)_i) + p 1_{(\theta(I)_i=1)} \eta_{E,i}^{\gamma(k)}(\gamma(l)_i) + (1-p) 1_{(\gamma(k)_i=\gamma(l)_i)}\big] + \varepsilon \nu_i(\gamma(l)_i) \Big\}, \tag{4}$$

where $1_{(\cdot)}$ is the indicator function. It is reasonable to assume that as time passes firms learn to use the rule of thumb given by exemplary or profit imitation ever better. Hence, the situation where the probability of mistakes ε converges to zero will be analyzed. That is, the Markov chain with transition matrix

$$M^I := \lim_{\varepsilon \downarrow 0} M_\varepsilon^I \tag{5}$$

is considered. Since M^I is an irreducible matrix, this Markov chain is ergodic and thus has a unique invariant probability measure $\mu^I(\cdot)$. For further reference the k-th element of the invariant measure will be denoted by μ_k^I, i.e. $\mu_k^I = \mu^I(\gamma(k))$. For technical reasons it is assumed that the distributions $\nu_i(\cdot)$ are such that all elementary divisors of the matrices M^I, $I = 1, \ldots, N$, are linear.

2.2 The behavioural level

It is reasonable to assume that each firm knows *ex ante* that cooperation yields higher profits than competition. Since anti-trust laws forbid explicit coordination the question is whether there can be non-cooperative coordination on the cooperative outcome. Behavioural considerations are modelled by using reciprocal arguments.[1] It is assumed that each firm can deduce the behavioural type of the other firms given their output choices. If a firm acts cooperatively and sees that its example is followed it will not change its behaviour. If however, cooperative behaviour does not give enough profit relative to competitive behaviour, the firm will change its behaviour to competition. To make the decision on which behaviour to choose firm i looks at the average profits resulting from competitive and cooperative behaviour. For each $I = 1, \ldots, N$ let the set of firms behaving cooperatively be denoted by $B(I)$, i.e. $B(I) = \{i \in I_n | \theta(I)_i = 1\}$, with cardinality $b(I) := |B(I)|$. Since all firms behaving cooperatively are exemplary imitators and all competitives are profit imitators, the average profits of these types of behaviour, given that output vector $\gamma(k)$ is played, are given by

$$U_E^k(I) = 1_{(b(I)>0)}\big[\frac{1}{b(I)} \sum_{j \in B(I)} \pi(\gamma(k)_j, \gamma(k)_{-j})\big] \tag{6}$$

and

$$U_P^k(I) = 1_{(b(I)<n)}\big[\frac{1}{n-b(I)} \sum_{j \in I_n \setminus B(I)} \pi(\gamma(k)_j, \gamma(k)_{-j})\big], \tag{7}$$

respectively.

Since full cooperation yields more profits than anything else, it is assumed that within the group of firms there is a certain inclination to cooperative behaviour. This inclination can be seen as a social contract between the members of the group. In this paper we are not concerned about the evolution of this contract but simply assume that it is reflected by an exogenous and constant parameter $\tilde{\lambda} > 0$. A firm will then behave cooperatively as long as the average profit of behaving competitively does not exceed the average profit of behaving cooperatively with $\tilde{\lambda}$.[2] So, given a permutation I on Θ^n and a permutation k on $\Gamma(\delta)^n$, the behaviour that firm i will choose

[1] In evolutionary economics reciprocity is regarded to be extremely important, see e.g. Binmore (1998).

[2] This behaviour may seem like a *deus ex machina*. However, since firms know that cooperation in the end yields more profits than competition, the parameter $\tilde{\lambda}$ can also be seen as the patience of the firms or the maximum amount of profit that they are willing to forgo to reach a cooperative outcome.

if it can adapt its behaviour is given by

$$\theta^k(I)_i = \begin{cases} 1 & \text{if } U_P^k(I) - U_E^k(I) < \tilde{\lambda} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Like in the quantity setting level, it is assumed that each period a firm adapts its behaviour with probability $\tilde{p} > 0$. Together with eq. (8) this yields a dynamic process that defines a Markov chain of pure behavioural adaptation dynamics.[3] It is assumed that with a positive probability $\tilde{\varepsilon} > 0$ each firm makes a mistake or experiments and chooses any behaviour using a probability measure $\tilde{\nu}_i(\cdot)$ with full support. Given a vector of quantities $\gamma(k) \in \Gamma(\delta)^n$, the probability of changing from $\theta(I)$ to $\theta(J)$ is then given by

$$\begin{aligned} \lambda_{\tilde{\varepsilon}}^k(I,J) = \prod_{i \in I_n} \Big\{ &(1-\tilde{\varepsilon}) \left[\tilde{p} 1_{(\theta(J)_i = \theta^k(I)_i)} + (1-\tilde{p}) 1_{(\theta(J)_i = \theta(I)_i)} \right] \\ &+ \tilde{\varepsilon} \tilde{\nu}_i(\theta(J)_i) \Big\}. \end{aligned} \tag{9}$$

As before, it is assumed that as time passes, firms get a better grip on the market situation and therefore the analysis is restricted to the dynamics where the experimentation probability converges to zero. To that end for each $k = 1, \ldots, m$ and $I, J = 1, \ldots, N$ define

$$\lambda_{IJ}^k := \lim_{\tilde{\varepsilon} \downarrow 0} \lambda_{\tilde{\varepsilon}}^k(I,J). \tag{10}$$

So, we consider the situation where evolution has forced the probability of mistakes on both the quantity setting level and the behavioural level to be infinitely small, but strictly positive.

To complete the model, we construct a Markov chain on Ω whose transition matrix is denoted by Q. At the beginning of each period firms simultaneously adapt their behaviour and output level based on the present state

[3]The postulated types of behaviour might give the impression that exemplary imitating implies solving a profit maximization problem, whereas profit imitation does not. Therefore it may seem that switching from cooperative to competitive behaviour implies loosing rationality, since exemplary imitators are more sophisticated. However, profit imitation can in fact also be seen as a form of profit maximization. The only difference is that it requires less computational burden. Therefore, switching from cooperative to competitive behaviour merely implies that the firm is not willing any more to endure a heavier computational burden for behaviour that it considers not to give the desired result.

of the world. A typical element of Q is then given by

$$Q_{k_I l_J} = \lambda^k_{IJ} M^I(k,l). \tag{11}$$

Note that Q is a stochastic and irreducible matrix. The matrix Q is constructed by making blocks of all permutations on Θ^n. Within these blocks all permutations on $\Gamma(\delta)^n$ are stored. Hence, the coordinate k_I implies the quantity vector $\gamma(k)$ in block I, i.e. with behaviour vector $\theta(I)$. Since Q is irreducible the Markov chain has a unique invariant distribution which is denoted by $\mu(\cdot)$. For technical convenience it is assumed that the $\tilde{\nu}_i(\cdot)$ are such that all elementary divisors of Q are linear.

To facilitate the analysis in the next section define the stochastic and irreducible matrix Q^* with typical element

$$Q^*_{k_I l_J} = \begin{cases} M^I(k,l) & \text{if } I = J \\ 0 & \text{if } I \neq J. \end{cases} \tag{12}$$

Note that Q^* is a block diagonal matrix with blocks $Q^*_I = M^I$, $I = 1, \ldots, N$. Furthermore, a scalar ζ and a matrix C are defined such that

$$\zeta C_{k_I l_J} = \begin{cases} \lambda^k_{IJ} M^I(k,l) & \text{if } I \neq J \\ -\sum_{J \neq I} \lambda^k_{IJ} M^I(k,l) & \text{if } I = J, \end{cases} \tag{13}$$

and

$$\begin{aligned} \zeta &= \max_{k_I} \Big\{ \sum_{J \neq I} \sum_{l=1}^{m} Q_{k_I l_J} \Big\} \\ &= \max_{k_I} \Big\{ \sum_{J \neq I} \lambda^k_{IJ} \Big\}. \end{aligned} \tag{14}$$

Note that $Q = Q^* + \zeta C$. For the results to be obtained in Section 3 to hold, the interaction between the blocks Q^*_I should be relatively small. It is most natural to achieve this by making assumptions on the probability of behavioural revision $\tilde{p}$, i.e. it is assumed that $\tilde{p}$ is small enough such that

$$\zeta < \tfrac{1}{2} \Big[1 - \max_{I=1,\ldots,N} |\lambda_2(Q^*_I)| \Big], \tag{15}$$

where $\lambda_2(Q^*_I)$ is the second largest eigenvalue of Q^*_I in absolute value.

3 Analysis

In this section we prove that in the long-run the system settles down either in the Walrasian equilibrium or in the monopoly outcome. To do so we need some additional notation.

Define the collusion outcome q^m to be the quantity that maximizes industry profits. That is, q^m satisfies

$$P(nq^m)q^m - C(q^m) \geq P(nq)q - C(q) \qquad \forall q \in \mathbb{R}_+. \tag{16}$$

Let q_r^*, $r = 0, 1, \dots, n$, be defined by

$$\begin{aligned} P((n-r)q_r^* + &rq^m)q_r^* - C(q_r^*) \\ &\geq P((n-r)q_r^* + rq^m)q - C(q) \qquad \forall q \in \mathbb{R}_+. \end{aligned} \tag{17}$$

Furthermore, let I_0 and I_1 be such that $\theta(I_0) = (0, \dots, 0)$ and $\theta(I_1) = (1, \dots, 1)$. Note that $q_n^* = q^m$ and that q_0^* is the Walrasian output level q^*. It is assumed that the finite grid is such that $q_r^* \in \Gamma(\delta)^n$ for all $r = 0, 1, \dots, n$.

On the quantity setting level, denote the monomorphic state $(q, \dots, q) \in \Gamma(\delta)^n$ by $\overline{\gamma}(q)$. For $\theta(I) \in \Theta^n$, the pseudo-monomorphic state $\overline{\gamma}_I(q, q') \in \Gamma(\delta)^n$ is defined by

$$\overline{\gamma}_I(q, q')_i = \begin{cases} q & \text{if } \theta(I)_i = 0 \\ q' & \text{if } \theta(I)_i = 1, \end{cases} \tag{18}$$

for all $i \in I_n$. Furthermore, k_I^* is defined to be the permutation on $\Gamma(\delta)^n$ such that $\gamma(k_I^*) = \overline{\gamma}_I(q_{b(I)}^*, q^m)$.

The first proposition that is proved determines the limit distribution on the quantity setting level for a Markov chain with transition matrix M^I, $I = 1, \dots, N$, i.e. the limit distribution of block Q_I^* is determined. To prove the proposition two lemmas are needed, the proofs of which can be found in the appendix. To simplify notation, for each $I = 1, \dots, N$ denote

$$\begin{aligned} \mathcal{A}_I = &\{\{\overline{\gamma}(q)\} | q \in \Gamma(\delta)\} \cup \Big\{\{\overline{\gamma}(q, q')\} | q, q' \in \Gamma(\delta), \\ &\pi(q, (n - b(I) - 1)q + b(I)q') > \pi(q', (n - b(I))q + (b(I) - 1)q'), \\ &\pi(q', (n-1)q') > \pi(q, (n-1)q')\Big\}. \end{aligned} \tag{19}$$

The first lemma determines the set of recurrent classes for the imitation dynamics.

Lemma 3.1 *Let $\theta(I) \in \Theta^n$ be a behaviour vector. The set of recurrent classes for the imitation dynamics with transition matrix M_0^I is given by $\mathcal{A}_I$.*

The next lemma is an extension of the claim in Vega-Redondo (1997, p. 381).

Lemma 3.2 *Let $\theta(I) \in \Theta^n$ be a behaviour vector.*
$\forall_{k \in \{1,2,\ldots,n-b(I)-1\}} \forall_{q \neq q^*_{b(I)}}$:

$$\begin{aligned} P(kq^*_{b(I)} + &(n-b(I)-k)q + b(I)q^m)q^*_{b(I)} - C(q^*_{b(I)}) \\ &> P(kq^*_{b(I)} + (n-b(I)-k)q + b(I)q^m)q - C(q). \end{aligned} \tag{20}$$

The first propostion can now be stated as follows.

Proposition 3.1 *Let $\theta(I) \in \Theta^n$ be a behaviour vector. The limit distribution μ^I of the Markov chain with transition matrix M^I is a well-defined element of $\Delta(\Gamma(\delta)^n)$. Moreover, $\mu^I_{k^*_I} = 1$.*

Proof. The theory used to prove the proposition is based on Freidlin and Wentzell (1984). See e.g. Kandori et al. (1993), Young (1998) or Vega-Redondo (1997) for a concise treatment of the theory. In the following we use the idea of costs between two states $\gamma(k)$ and $\gamma(l)$. Let $d(\gamma(k), \gamma(l)) = \sum_{i \in I_n} 1_{(\gamma(k)_i \neq \gamma(l)_i)}$. The costs between $\gamma(k)$ and $\gamma(l)$ are then defined by

$$c(\gamma(k), \gamma(l)) = \min_{j=1,\ldots,m} \left\{ d(\gamma(k), \gamma(j)) | M_0^I(j,l) > 0 \right\},$$

i.e. $c(\gamma(k), \gamma(l))$ gives the minimum number of mutations from $\gamma(k)$ that is needed for the imitation dynamics to have positive probability of reaching $\gamma(l)$. In the remainder the argument of $\gamma(\cdot)$ is supressed.

First, we build a $\overline{\gamma}_I(q^*_{b(I)}, q^m)$-tree H^*, with minimal costs. Then it is shown that for any other state $\gamma \neq \overline{\gamma}_I(q^*_{b(I)}, q^m)$ and any γ-tree H_γ the costs will be higher. Young (1993) has shown that the minimum cost tree is among the γ-trees where γ is an element of a recursive class of the pure imitation dynamics. Thus, following Lemma 3.1 only the elements of $\mathcal{A}_I$ need to be considered. So, take $\{\gamma\} \in \mathcal{A}_I$. There are four possibilities.

1. $\gamma = \overline{\gamma}(q)$, $q \neq q^*_{b(I)}$, $q \neq q^m$. This case can be subdivided in two parts.

(a) $\pi(q,(n-b(I)-1)q+b(I)q^m) > \pi(q^m,(n-b(I))q+b(I)q^m)$. Consider the following sequence of events.

i. One exemplary imitator mutates to q^m, denoting the resulting state by γ'. This gives cost $c(\gamma,\gamma')=1$.

ii. All exemplary imitators get the opportunity to revise their output. They all choose q^m with positive probability.

iii. All profit imitators may adapt their output level. They all stay at q with positive probability.

So, the state $\overline{\gamma}(q,q^m) \in \mathcal{A}_I$ is reached at cost 1.

(b) $\pi(q,(n-b(I)-1)q+b(I)q^m) < \pi(q^m,(n-b(I))q+(b(I)-1)q^m)$. It takes one mutation of an exemplary imitator to q^m to reach with positive probability the state $\overline{\gamma}(q^m) \in \mathcal{A}_I$, by revisions of both exemplary and profit imitators. Hence $c(\gamma,\overline{\gamma}(q^m))=1$.

2. $\gamma=\overline{\gamma}(q,q')$, $q,q' \neq q^*_{b(I)}$, $q,q' \neq q^m$. This case too can be subdivided in two parts.

(a) $\pi(q,(n-b(I)-1)q+b(I)q^m) > \pi(q^m,(n-b(I))q+(b(I)-1)q^m)$. As before it takes one mutation to reach $\overline{\gamma}(q,q^m)$ with positive probability.

(b) $\pi(q,(n-b(I)-1)q+b(I)q^m) < \pi(q^m,(n-b(I))q+(b(I)-1)q^m)$. State $\overline{\gamma}(q^m)$ can be reached with positive probability using one mutation of an exemplary imitator.

3. $\gamma=\overline{\gamma}(q^*_{b(I)},q)$, $q \neq q^m$. It takes one mutation of an exemplary imitator to reach $\overline{\gamma}_I(q^*_{b(I)},q^m)$ with posititve probability.

4. $\gamma=\overline{\gamma}(q,q^m)$, $q \neq q^*_{b(I)}$.[4] It takes one mutation of a profit imitator to $q^*_{b(I)}$ to reach $\overline{\gamma}_I(q^*_{b(I)},q^m)$ with positive probability. If one profit imitator mutates to $q^*_{b(I)}$ and next period all profit imitators get the opportunity to revise their production then, by applying Lemma 3.2 with $k=1$, one can see that all profit imitators will choose $q^*_{b(I)}$ with positive probability, thus reaching $\overline{\gamma}_I(q^*_{b(I)},q^m)$.

[4]Note that $q=q^m$ also belongs to this case.

By tying all states of the form of case 1 and 2 to states of the form of case 4 and tying all states of the form of cases 3 and 4 to $\overline{\gamma}_I(q^*_{b(I)}, q^m)$, we have constructed a $\overline{\gamma}_I(q^*_{b(I)}, q^m)$-tree H^* with $c(H^*) = |\mathcal{A}_I| - 1$.

As shown above, to leave any state $\gamma \in \mathcal{A}_I \backslash \{\overline{\gamma}_I(q^*_{b(I)}, q^m)\}$ it takes at least one mutation. Consider now $\overline{\gamma}_I(q^*_{b(I)}, q^m)$. Suppose that for two periods to come, no exemplary imitator can revise its output. If one profit imitator mutates to, say, q and all profit imitators afterwards get the opportunity to revise their output, then by applying Lemma 3.2 with $k = n - b(I) - 1$, one can see that all profit imitators will choose $q^*_{b(I)}$. So, it takes at least two mutations to leave $\overline{\gamma}_I(q^*_{b(I)}, q^m)$. Hence for every $\gamma \in \mathcal{A}_I \backslash \{\overline{\gamma}_I(q^*_{b(I)}, q^m)\}$ and any γ-tree H_γ we have

$$c(H_\gamma) > |\mathcal{A}_I| - 1 = c(H^*). \tag{21}$$

Thus, $\mu^I(\overline{\gamma}_I(q^*_{b(I)}, q^m)) = \mu^I_{k^*_I} = 1$.

$\square$

To state the proposition on the invariant distribution of the entire dynamics with transition matrix Q let us first introduce some more notation. Suppose that there are r firms behaving cooperatively and hence, $n-r$ firms that are behaving competitively. Then denote by $\underline{B}$ the set consisting of the values that r can have so that the average profit of profit imitation exceeds the average profit of exemplary imitating by not more than $\tilde{\lambda}$, i.e.

$$\begin{aligned}\underline{B} = \{r \in \{0, 1, \ldots, n\} | \pi(q^*_r, (r-1)q^*_r + (n-r)q^m) \\ - \pi(q^m_r, rq^*_r + (n-r-1)q^m) < \tilde{\lambda}\}.\end{aligned} \tag{22}$$

Furthermore, we define $\overline{B} := \{0, 1, \ldots, n\} \backslash \underline{B}$. The minimum number of cooperative firms that needs to be present in the market to make cooperation possible for the entire market is then given by

$$r_0 = \begin{cases} \min\{r | r \in \underline{B}\} & \text{if } \underline{B} \neq \emptyset \\ n & \text{if } \underline{B} = \emptyset. \end{cases} \tag{23}$$

Similarly, the minimum number of competitive firms needed to make competition feasible for the entire market is given by

$$r_1 = \begin{cases} \min\{r | r \in \overline{B}\} & \text{if } \overline{B} \neq \emptyset \\ n & \text{if } \overline{B} = \emptyset. \end{cases} \tag{24}$$

Consider the Markov chain on $\Theta^n \times \Theta^n$ with transition matrix $\tilde{M}$, a typical element of which is given by

$$
\begin{aligned}
p_{IJ} &= \sum_{k=1}^{m} \mu_k^I \sum_{l=1}^{m} Q_{k_I l_J} \\
&= \sum_{k=1}^{m} \mu_k^I \lambda_{IJ}^k \sum_{l=1}^{m} M^I(k,l) \\
&= \sum_{k=1}^{m} \mu_k^I \lambda_{IJ}^k = \lambda_{IJ}^{k_I^*}.
\end{aligned} \tag{25}
$$

Note that the matrix $\tilde{M}$ is an aggregate version of Q where aggregation has taken place over the quantity setting level. Furthermore, remark that $\lambda_{IJ}^{k_I^*}$ is the limit of a pure behavioural adaptation process and a mutation process where the probability of a mutation converges to zero. Let us first consider the pure behavioural adaptation process.

Lemma 3.3 *The set of recurrent states for the pure behavioural adaptation dynamics is given by* $\tilde{\mathcal{A}} = \{\theta(I_0), \theta(I_1)\}$.

The following proposition can now be stated.

Proposition 3.2 *Assume that* $U_P^{k_{I_0}^*} \geq \tilde{\lambda}$. *The unique invariant probability measure* $\mu(\cdot)$ *can be approximated by a well defined element* $\tilde{\mu}(\cdot)$ *of* $\Delta(\Omega)$. *For* $\tilde{\mu}(\cdot)$ *it holds that*

1. *If* $r_0 < r_1$, *then* $\tilde{\mu}_{k_{I_1}^*} = 1$.
2. *If* $r_0 > r_1$, *then* $\tilde{\mu}_{k_{I_0}^*} = 1$.
3. *If* $r_0 = r_1$, *then* $\tilde{\mu}_{k_{I_0}^*} = \tilde{\mu}_{k_{I_1}^*} = 0.5$.

Furthermore, the approximation $\tilde{\mu}(\cdot)$ *of* $\mu(\cdot)$ *is of order* $O(\zeta)$.

Proof. Since $\tilde{M}$ is an irreducible matrix there exists a unique invariant probability measure $\tilde{\mu}(\cdot)$. The procedure to prove the proposition is the same as in the proof of Proposition 3.1, i.e. we build θ-trees and the ones with minimal costs get positive measure. Since only the elements of $\tilde{\mathcal{A}}$ are to be considered we only need to compare $c(\theta(I_0), \theta(I_1))$ and $c(\theta(I_1), \theta(I_0))$. Suppose that $r_0 < r_1$. Then $c(\theta(I_0), \theta(I_1)) = r_0 < r_1 = c(\theta(I_1), \theta(I_0))$. Since we only need to consider the stochastically stable states from the quantity setting level, we conclude that $\tilde{\mu}_{k^*_{I_1}} = 1$. The other cases are proved in a similar way.

Since it is assumed that eq. (15) holds and that the elementary divisors of Q and Q^*_I, $I = 1, \ldots, N$ are linear, the matrix Q is nearly completely decomposable. Following Courtois (1977) we can conclude that the aggregation in eq. (25) leads to an approximation of the invariant measure of Q that is of order $O(\zeta)$.

□

The condition $U_P^{k^*_{I_0}} \geq \tilde{\lambda}$ in Proposition 3.2 is not necessary, for if it does not hold, then the only recurrent state in the pure behavioural adaptation process is $\theta(I_1)$ which would lead immediately to the conclusion that this is the unique stochastically stable state. It is however reasonable to assume that coordination on the competitive outcome is feasible.

4 Conclusion

In this paper we extended the model of Vega-Redondo (1997) in two ways. Multiple behavioural rules are allowed and behavioural adaptation is introduced. This set-up leads to the conclusion that either competition or cooperation is stochastically stable (unless $r_0 = r_1$, in which case both are stochastically stable), depending on the structure of the market and the inclination to cooperative behaviour.[5] Competition leads to the Walrasian equilibrium and cooperation leads to the collusion equilibrium. This paper can therefore be seen as providing a theoretical underpinning for some of the experimental results found by Offerman et al. (2000).

In their experiment Offerman et al. (2000) randomly match individuals into groups of three persons. Each person is given the inverse demand function and the cost function. Individuals do not know to whom they are matched. In each round, each individual chooses an output level given the information that she gets. Offerman et al. (2000) use three informational treatments. The one that corresponds to our model is the treatment in

[5] Recall that the inclination to cooperative behaviour can be interpreted as the patience that firms have. Therefore, the inclination to cooperative behaviour has the same role in this model as the discount rate in the infinitely repeated Cournot game. For a concise treatment of the infinitely repeated Cournot game see e.g. Tirole (1988).

which after each round every individual is informed about the revenues, costs, profits, and quantities of all firms in her group. Furthermore, the aggregate quantity as well as the market price of each person's group is given. Following Vega-Redondo (1997) one would expect that every group eventually settles at the Walrasian equilibrium. However, this is not the case. The collusion equilibrium is also reached quite often in this experiment. According to the theory presented in this paper, the co-existence of both equilibria is the result of different levels of the inclination to cooperative behaviour in groups.

It is a natural further step to drop the assumption of a common inclination to cooperative behaviour and replace it with individual levels. This could then facilitate an analysis into the impact of these levels on the resulting market outcome. Furthermore, the model might be extended with more behavioural rules. Offerman et al. (2000) for instance pose several other plausible behavioural rules. In this perspective, it would be interesting to distinguish between behavioural rules by attaching costs of using the rules, for instance based on the level of information that is available. If lots of information is available, profit imitation is a feasible behavioural rule that does not require much intellectual or computational skills. Therefore, it can be seen as a cheap behavioural rule. If only aggregate quantities are known, profit imitation is not feasible any more. Fictitious play for instance, still is. However, fictitious play requires more skills and can therefore be regarded as more expensive. It is conjectured by Offerman et al. (2000) that these costs are influencing the resulting equilibria in different information treatments.

The results from this model justify the attention that anti-trust agencies have for so-called *concerted practice*.[6] This legal term comprises nothing more than market coordination between firms without making legally binding agreements, i.e. playing the collusion equilibrium. To determine whether concerted practice is feasible using the theory in this paper, the anti-trust agency must have insight in the inclination to cooperative behaviour. This may seem impossible, but intensity of competition might be a good estimate. For a discussion on several measures for competition we refer to Boone (2000).

[6]That is, if one believes that collusion is bad for welfare, which is a generally accepted viewpoint among neo-classical economists.

Finally, for anti-trust agencies to make a good analysis of a given market, entry and exit have to be taken into account. Therefore, it would seem appropriate to extend the model by accomodating for entry and exit. In fact, Alós-Ferrer et al. (1999) extended the Vega-Redondo (1997) model in this direction, without the possibility of behavioural adaptation however.

Appendix

A Proof of Lemma 3.1

Note that every element of $\mathcal{A}_I$ is a recurrent class. We prove the converse claim. Let $\gamma \in \Gamma(\delta)^n$ be such that there is no $A \in \mathcal{A}_I$ satisfying $\gamma \in A$. We will show that there is a path from γ to a state in $\mathcal{A}_I$ with positive probability.

Take $q \in \{\gamma_j | \pi(\gamma_j, \gamma_{-j}) = \max_{i \in I_n} \{\pi(\gamma_i, \gamma_{-i})\}$ and denote any exemplary choice in $\Gamma(\delta)^n$ by q'. With positive probability all profit imitators choose q and all exemplary imitators choose q' next period. So, next period's state will be $\overline{\gamma}(q, q')$. There are two possibilities.

1. $q = q'$; then $\{\overline{\gamma}(q, q')\} = \{\overline{\gamma}(q)\} \in \mathcal{A}_I$.

2. $q \neq q'$. There are two cases[7].

   (a) $\pi(q, (n - b(I) - 1)q + b(I)q') \geq \pi(q', (n - b(I))q + (b(I) - 1)q')$ and $\pi(q', (n-1)q') \geq \pi(q, (n-1)q)$.
   Then it holds that

   $$q \in BP(\overline{\gamma}(q, q'))$$

   and

   $$q' \in BE(\overline{\gamma}(q, q')).$$

   So, with positive probability state $\overline{\gamma}(q, q')$ is reached and $\{\overline{\gamma}(q, q')\} \in \mathcal{A}_I$.

[7]Since q' is an exemplary choice, $\pi(q', (n-1)q') \geq \pi(q, (n-1)q)$.

(b) $\pi(q,(n-b(I)-1)q+b(I)q') < \pi(q',(n-b(I))q+(b(I)-1)q')$ and $\pi(q',(n-1)q') \geq \pi(q,(n-1)q)$.
With positive probability, all firms choose q' leading to the monomorphic state $\overline{\gamma}(q') \in \mathcal{A}_I$.

This proves the lemma.

□

B Proof of Lemma 3.2

Since $P(\cdot)$ is strictly decreasing we have for all $q \neq q^*_{b(I)}$

$$[P((n-b(I))q^*_{b(I)} + b(I)q^m) - P(kq^*_{b(I)} + (n-b(I)-k)q + b(I)q^m)]q^*_{b(I)} \\ < [P((n-b(I))q^*_{b(I)} + b(I)q^m) - P(kq^*_{b(I)} + (n-b(I)-k)q + b(I)q^m)]q. \tag{26}$$

Substracting $C(q) - C(q^*_{b(I)})$ from both sides and rearranging terms gives

$$[P((n-b(I))q^*_{b(I)} + b(I)q^m)q^*_{b(I)} - C(q^*_{b(I)})] + [P(kq^*_{b(I)} + (n-b(I)-k)q + \\ + b(I)q^m)q - C(q)] < [P((n-b(I))q^*_{b(I)} + b(I)q^m)q - C(q)] \\ + [P(kq^*_{b(I)} + (n-b(I)-k)q + b(I)q^m)q^*_{b(I)} - C(q^*_{b(I)})]. \tag{27}$$

From eq. (17) it can be seen that the first term on the left hand side is as least as large as the first term on the right hand side. This proves the lemma.

□

C Proof of Lemma 3.3

Note that $\tilde{M}(I,J) = \lambda^{k^*_I}_{IJ}$. Hence, we only need to consider the quantity vectors that are stochastically stable in the quantity setting level. Suppose the system is in $\theta(I_0)$. Then $U^{k^*_{I_0}}_P - U^{k^*_{I_0}}_E = U^{k^*_{I_0}}_P \geq \tilde{\lambda}$. Hence, $\theta^{k^*_{I_0}}_i = 0$ for all $i \in I_n$, and the system stays in $\theta(I_0)$.

Suppose the system is in $\theta(I_1)$. Then $U^{k^*_{I_1}}_P - U^{k^*_{I_1}}_E = -U^{k^*_{I_1}}_E < \tilde{\lambda}$. Hence, $\theta^{k^*_{I_1}}_i = 1$ for all $i \in I_n$, and the system stays in $\theta(I_1)$.

To prove the converse claim, let $\theta \in \Theta^n \backslash \tilde{\mathcal{A}}$. Suppose that all firms may change behaviour which happens with positive probability. Then all firms choose the same behaviour. So, the system reaches either $\theta(I_0)$ or $\theta(I_1)$. This proves the lemma.

$\square$

References

Alchian, A.A. (1950). Uncertainty, Evolution, and Economic Theory. *Journal of Political Economy*, **58**, 211–221.

Alós-Ferrer, C., A.B. Ania, and F. Vega-Redondo (1999). An Evolutionary Model of Market Structure. In: P.J.J. Herings, G. van der Laan, and A.J.J. Talman (Eds.), *The Theory of Markets*, pp. 139–163. North-Holland, Amsterdam.

Ando, A. and F.M. Fisher (1963). Near-Decomposability, Partition and Aggregation, and the Relevance of Stability Discussions. *International Economic Review*, **4**, 53–67.

Binmore, K. (1998). *Game Theory and the Social Contract, volume 2: Just Playing*. MIT-press, Cambridge, Mass.

Boone, J. (2000). *Competition*. CentER, DP 2000-104, Tilburg University, Tilburg.

Courtois, P.J. (1977). *Decomposability, Queueing and Computer System Applications*. ACM monograph series. Academic Press, New York, NY.

Foster, D.P. and H.P. Young (1990). Stochastic Evolutionary Game Dynamics. *Theoretical Population Biology*, **38**, 219–232.

Freidlin, M.I. and A.D. Wentzell (1984). *Random Perturbations of Dynamical Systems*. Springer-Verlag, Berlin.

Güth, W. and M. Yaari (1992). An Evolutionary Approach to Explain Reciprocal Behavior in a Simple Strategic Game. In: U. Witt (Ed.), *Explaining Process and Change - Approaches to Evolutionary Economics*. The University of Michigan Press.

Harsanyi, J. and R. Selten (1988). *A General Theory of Equilibrium Selection in Games*. MIT-press, Cambridge, Mass.

Kandori, M., G.J. Mailath, and R. Rob (1993). Learning, Mutation, and Long Run Equilibria in Games. *Econometrica*, **61**, 29–56.

Maynard Smith, J. (1982). *Evolution and the Theory of Games*. Cambridge University Press, Cambridge, UK.

Offerman, T., J. Potters, and J. Sonnemans (2000). *Imitation and Belief Learning in an Oligopoly Experiment*. mimeo, University of Amsterdam and Tilburg University.

Possajennikov, A. (2000). *Learning and Evolution in Games and Oligopoly Models*. Ph. D. thesis, CentER for Economic Research, Tilburg University, Tilburg.

Simon, H.A. and A. Ando (1961). Aggregation of Variables in Dynamic Systems. *Econometrica*, **29**, 111–138.

Tirole, J. (1988). *The Theory of Industrial Organization*. MIT-press, Cambridge, Mass.

van Damme, E.E.C. (1994). Evolutionary Game Theory. *European Economic Review*, **38**, 847–858.

Vega-Redondo, F. (1997). The Evolution of Walrasian Behavior. *Econometrica*, **65**, 375–384.

Weibull, J. (1995). *Evolutionary Game Theory*. MIT-press, Cambridge, Mass.

Young, H.P. (1993). The Evolution of Conventions. *Econometrica*, **61**, 57–84.

Young, H.P. (1998). *Individual Strategy and Social Structure*. Princeton University Press, Princeton, NJ.